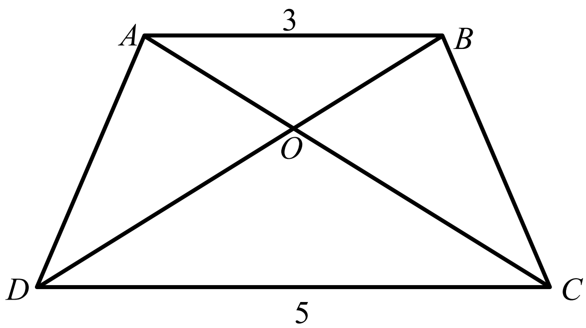
Hong Kong Mathematics Olympiad (2012 / 2013)
Heat Event (Group)
香港数学竞赛 (2012 / 2013)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知一个直角三角形三边的长度皆为整数，且其中两边的长度为方程 $x^2-(m+2)x+4m=0$ 的根。求第三边长度的最大值。
Given that the length of the three sides of a right-angled triangle are integers, and two of them are the roots of the equation $x^2-(m+2)x+4m=0$. Find the maximum length of the third side of the triangle.

2. 图一所示为一梯形 $ABCD$，其中 $AB=3$、$CD=5$ 及对角线 AC、BD 相交于点 O。若 ΔAOB 的面积是 27，求梯形 $ABCD$ 的面积。
Figure 1 shows a trapezium $ABCD$, where $AB=3$, $CD=5$ and the diagonals AC and BD meet at O. If the area of ΔAOB is 27, find the area of the trapezium $ABCD$.

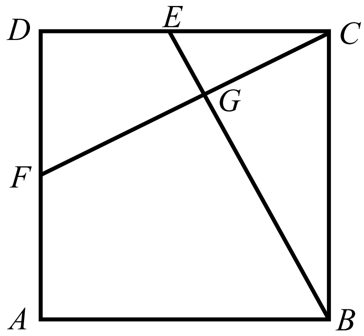


图一
Figure 1

3. 设 x 及 y 为实数使得 $x^2+xy+y^2=2013$。求 x^2-xy+y^2 的最大值。
Let x and y be real numbers such that $x^2+xy+y^2=2013$. Find the maximum value of x^2-xy+y^2.

4. 若 α、β 是方程 $x^2+2013x+5=0$ 的根，求 $(\alpha^2+2011\alpha+3)(\beta^2+2015\beta+7)$ 的值。
If α, β are roots of $x^2+2013x+5=0$, find the value of $(\alpha^2+2011\alpha+3)(\beta^2+2015\beta+7)$.

5. 如图二所示，$ABCD$ 为一个边长为 10 单位的正方形，E 及 F 分别为 CD 及 AD 的中点，BE 及 FC 相交于 G。求 AG 的长度。
As shown in Figure 2, $ABCD$ is a square of side 10 units, E and F are the mid-points of CD and AD respectively, BE and FC intersect at G. Find the length of AG.



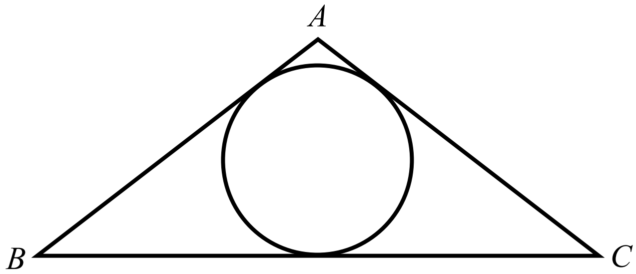
图二

Figure 2

6. 若 a 及 b 为正实数，且方程 $x^2+ax+2b=0$ 及 $x^2+2bx+a=0$ 都有实数根。求 $a+b$ 的最小值。
If a and b are positive real numbers, and the equations $x^2+ax+2b=0$ and $x^2+2bx+a=0$ have real roots. Find the minimum value of $a+b$.

7. 已知 ΔABC 的三边的长度组成一个等差数列，且为方程 $x^3-12x^2+47x-60=0$ 的根，求 ΔABC 的面积。
Given that the length of the three sides of ΔABC form an arithmetic sequence, and are the roots of the equation $x^3-12x^2+47x-60=0$, find the area of ΔABC.

8. 图三中，ΔABC 为一等腰三角形，其中 $AB=AC$，$BC=240$。已知 ΔABC 的内接圆的半径是 24，求 AB 的长度。
In Figure 3, ΔABC is an isosceles triangle with $AB=AC$, $BC=240$. The radius of the inscribed circle of ΔABC is 24. Find the length of AB.



图三

Figure 3

9. 从 1，2，3，…，2012，2013 中最多可取出多少个数，使得在取出的数中任意两个数之和都不是这两个数之差的倍数？

At most how many numbers can be taken from the set of integers : 1 , 2 , 3 , … , 2012 , 2013 such that the sum of any two numbers taken out from the set is not a multiple of the difference between the two numbers.

10. 对所有正整数 n，定义函数 f 为

(i) $f(1)=2012$，

(ii) $f(1)+f(2)+\cdots+f(n-1)+f(n)=n^2 f(n)$， $n>1$

求 $f(2012)$ 的值。

For all positive integers n, define a function f as

(i) $f(1)=2012$,

(ii) $f(1)+f(2)+\cdots+f(n-1)+f(n)=n^2 f(n)$, $n>1$

Find the value of $f(2012)$.

完

END